BSc Part III PAPER - VI Group A

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Centre of a Group

Def - Let G be a group. Let
$$Z(G) = \{x \in G \mid xg = gx \ \forall g \in G\}$$
Then $Z(G)$ is called centre of the Group G.

Th Centre of a group G is a subgroup of G.

Proof: Let $Z(G)$ be the centre of the group G.
Then $Z(G) \neq \varphi$ as $e \in Z(G)$

Again

$$x, y \in Z(G) \Rightarrow xg = gx$$
$$yg = gy \ \forall g \in G$$

$$\Rightarrow g^{-1}x^{-1} = x^{-1}g^{-1}$$
$$g^{-1}y^{-1} = y^{-1}g^{-1} \quad \forall g \in G$$

Now $g(xy^{-1}) = (gx)y^{-1} = (xg)y^{-1}$
$$= (xg)y^{-1}(g^{-1}g)$$

$= xg(y^{-1}g^{-1})g = xg(g^{-1}y^{-1})g$

$= x(gg^{-1})y^{-1}g$

$= (xy^{-1})g$ for all $g \in G$

$\Rightarrow xy^{-1} \in Z(G)$

Hence $Z(G)$ is a subgroup.